

External sources observed in a high curvature system

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Abstract: Many electromagnetic sources in our universe are extremely weak and complicated to be detected by our instruments, among the most well know low energy sources are for example the Cosmic Microwave Background radiation that comes from decoupling epoch, the light of the first stars in our universe expected from reionization epoch or even the Hawking radiation that it is expected that comes from black holes and impossible to be detected with our current technology. Is in this vein, that this paper is devoted to study those sources near of an extremely no collapsed high curvature system, presenting its blue-shift equations and its limits for the energy received.

Keywords: Black Holes, Gravitational Redshift.

1. Introduction

Recently, strong gravity (high curvature) systems like neutron stars (NS) or black holes (BH), even more exotic systems like boson [1] or quark stars [2], are trending topics in gravitation community. Furthermore, high curvature phenomena as collisions of NS-BH or BH-BH play an important role as sources of gravitational waves detected by the Laser Interferometer Gravitational-Waves Observatory (LIGO) and VIRGO experiments [3], being the first evidences of their existence. Later, the BH existence was confirmed directly through the shadow observation of the super-massive BH in the center of the galaxy Messier 87 (M87*) [4]. Motivated by these discoveries, the community is taking more interest in understand the nature and evolution of the accretion disks surround NS and BH, using advanced computing tools to solve the highly nonlinear equations presented in the relativistic hydrodynamics formulation [5, 6]. From the mathematical-physics point of view, the theorems that predicts the unavoidable singularity [7] are also useful to understand the fate of matter after the collapse. Some interesting studies are related to the effects produced when an observer is in an extremely no collapsed high curvature system and observe photons coming from outside, emitted for example, by stars or other radiation sources such as the cosmic microwave background (CMB) [8] or from the reionization epoch [9]. In literature, it is studied how the photons that emerges from a gravitational field are red-shifted, measured in experiments like that developed by Pound and Rebka [10]. Therefore, the blueshift is also possible and computed straightforward in Schwarzschild symmetry.

However, it is not extensively discussed how an extreme curvature system is bathed by high energy photons of diverse sources, no matter if those sources are weaker like these mentioned previously.

In this vein, we present the coordinate system that we will use, and with that, we show the related equation of the receiver in terms of the energy from a distant source subdued to a Schwarzschild symmetry. Hence, we explore how the radiation of an external source increase its energy in a high curvature system trending to infinity in the extreme case of the Schwarzschild radius. We focus our attention in study and discuss three phenomena: First, the cosmological photons that comes from the CMB radiation [8] and subsequently, the T_{21} radiation that comes from reionization epoch [11]. Both radiations are considered to be not coupled to the Hubble flux (global effects), only interacting with a Schwarzschild space-time. The third scenario is through a source of a BH partner expelling Hawking radiation [12] and observed in the extreme curvature system, indeed, in this case the Hawking radiation suffers an increase of energy (blue-shift) due to the curvature of the system where it is received.

The paper outline is structured as follows. Section II describes the blue shift effect when an observer is inside of a massive and extreme curvature system. In particular, we study situations where low energy photons like those coming from CMB radiation, T_{21} , or Hawking radiation. In Sec. III, we give our discussions and conclusions. We henceforth that we use units in which $c = \hbar = k_B = 1$.

2. The experiment

We start this analysis, with a *gedankenexperiment*, positioning ourselves on a massive, static and spherical symmetric system in Schwarzschild coordinates¹, calling ourselves as the *receivers*; hence the line element is given by the equation:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (1)$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2$ is the solid angle, M is the mass of the object² and G is the Newton gravitational constant with units of $[m_p]^{-2} = [eV]^{-2}$, being m_p the Planck mass.

The impact parameter allows that the photons from the *source* enters directly in the gravitational influence zone of the system, gaining linear momentum while the photons are getting closer to the receiver and tending to infinity energy for the limit region in where $R = 2GM$. This effect is well understood and described by the equation used previously in the Pound and Rebka experiment [10], for the red/blue shift as:

$$\nu_r = \left[\left(1 - \frac{2GM}{R + d_s}\right) \left(1 - \frac{2GM}{R}\right)^{-1} \right]^{\frac{1}{2}} \nu_s, \quad (2)$$

where $\nu_r(\nu_s)$ are the frequency of the receiver and the source respectively, d_s is the distance from the receiver to the source and the radial parameter is fixed to R, being now the object radius. For instance, we are considering that the velocity of the source is less compared to the light velocity, $\nu_s \ll 1$.

In the Pound and Rebka experiment we have that $d_s \ll R$; however, for our case the distance of the source (stars, CMB, light from reionization era, etc.) are far away from the system radius $d_s \gg R$, hence our previous equation can be rewritten as:

$$E_r \approx \left(1 - \frac{2GM}{R}\right)^{-\frac{1}{2}} E_s, \quad (3)$$

where now, we write the equation in terms of the energy of the receiver and the source. For example, the present energy of a CMB photon, far from a strong gravitational field or other interaction is $\sim 2.36 \times 10^{-4} eV$, but when we are tending to a Schwarzschild space-time in particular, to the event-horizon, ($R \rightarrow 2GM$), the energy of the CMB photon tends to infinity ($E_r \rightarrow \infty$). The extreme case $R = 2GM$ is known as BH (collapsed system) and the coordinates used here are not appropriate for this limit. Moreover, a no collapsed system is ruled by the relation $GM/R < 4/9$ [13], being the limit case $GM/R = 4/9$. Indeed, a BH can be considered as a collapsed system because its frontier is given by the event horizon in where $GM/R = 1/2$.

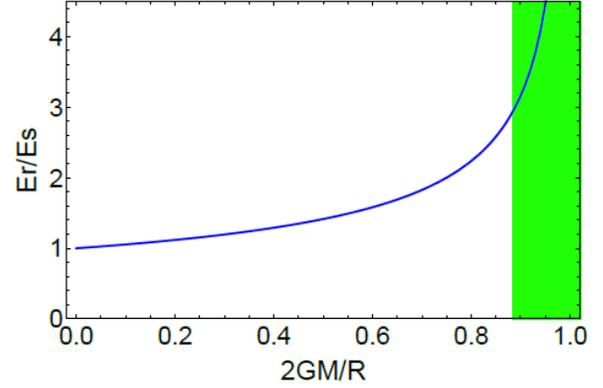


Fig. 1. The figure shows an increase of the ratio E_r / E_s while the observer is getting closer to the Schwarzschild radius, given by the expression $2GM/R \rightarrow 1$. The forbidden region is marked by the green contour, in where we are dealing with a collapsed system.

Eq. (3), behavior is depicted in Fig. 1, where it is possible to observe that when $2GM/R \rightarrow 1$, the ratio $E_r / E_s \rightarrow \infty$. However, exist a forbidden region marked by the green contour, where the object is a collapsed system, the limit region is dictated by the Schwarzschild radius ($2GM/R \rightarrow 1$) in where photons from the exterior are trapped and will inevitably fall to the singularity. Hence, avoiding collapsed systems, we have a maximum of the ratio $E_r / E_s = 3$ and it is when $2GM/R = 8/9$.

2.1 CMB and T_{21} photons

Weak photons, for example, those that comes from CMB or T_{21} are increased, even existing the possibility to be detected without a strong effort in a high curvature system surroundings, but of course the other sources create a background noise with increased energy also. In case of no collapsed systems with high curvature, this detection could be no so easy, due that the energy in the receptor is only three times the energy of the source as we show previously (avoiding other extra effects). In particular for CMB radiation, the expression is given by the equation:

$$E_r^{CMB} \approx \left(1 - \frac{2GM}{R}\right)^{-\frac{1}{2}} E_0^{CMB} (z_f + 1), \quad (4)$$

where E_r^{CMB} is the present energy of the CMB radiation ($2.36 \times 10^{-4} eV$) and z_f is a fixed redshift in where the photons are coupled with the Schwarzschild space-time. In this fixed region, the CMB photons are not coupled to the expansion of the Universe.

On the other hand, for the case of the T_{21} temperature, the energy measured in the system with Schwarzschild curvature is:

¹ Schwarzschild coordinates are not appropriate to study regions where $R \leq 2GM$, i.e., collapsed regions like a BH.

² Despite the Schwarzschild metric is useful for any mass, in this case we expect that the masses studied are compared at least with NS masses.

$$E_r^{T_{21}} \approx \left(1 - \frac{2GM}{R}\right)^{-\frac{1}{2}} \left(\frac{A_{10}\eta_{HI}(z_f)}{8\nu_0^2 H_0(z_f + 1)}\right) E(z_f)^{-1}, \quad (5)$$

being A_{10} the emission coefficient for the hyperfine triple singlet transition, $\eta_{HI}(z_f) = \eta_0(z_f + 1)^3$ called the number density of neutral Hydrogen, being $\eta_0 = (1 - Y)\rho_c / m_H$, where Y is the Helium mass fraction, ρ_c is the critical density in cosmology, m_H is the Hydrogen mass, ν_0 is the rest frame transition frequency, H_0 is the Hubble constant at $z = 0$ and $E(z_f)^2 \equiv (H(z_f) / H_0)^2 = \Omega_{or}(z_f + 1)^4 + \Omega_{om}(z_f + 1)^3 + \Omega_{o\Lambda}$, is the assumed Λ CDM cosmology for a at Universe, being Ω_{oi} the density parameters of radiation, matter and cosmological constant respectively. In similarity with CMB radiation, we are considering that T_{21} is not coupled to the Hubble flux and it is elected some fixed region z_f , where the observer is positioned; notice that the present time is considered at $z_f = 0$.

2.2 Hawking photons

Another interesting phenomena that is importantly increased in the region of a Schwarzschild space-time, are the photons that comes from a BH, known as the Hawking radiation [12]. In this scenario, the Hawking radiation blue-shift takes the form:

$$E_r^H \approx \left(1 - \frac{2GM}{R}\right)^{-\frac{1}{2}} (8\pi G m_{s(BH)})^{-1}, \quad (6)$$

where $m_{s(BH)}$ is the mass of the source, which in this case should be a BH mass that it is emitting the Hawking radiation.

A binary system of BH together with a no collapsed system, could observe this effect, the Hawking radiation from the BH source ($m_{s(BH)}$) is increasingly observed in the system partner (M) as Eq. (6) dictates at least assuming a Schwarzschild symmetry and the distance of the partner is larger enough compared with the radius of the BH.

In order to explore different cases for the potential GM/R , we define the following dimensionless variables $E_r \equiv \sqrt{G} E_r$ and $X \equiv 8\pi \sqrt{G} m_{s(BH)}$. In Fig. 2 we show the increase of energy for different masses of the source, moreover, it is presented the limit case and the forbidden region, where $GM/R = 4/9$. Notice how as we expected, the energy of the Hawking radiation increases while the system tends to a smaller BH. Moreover, we have a sustainable increase of energy when the potential energy (GM/R) of the receiver is close to the limit of a collapsed system marked by the continuous red line. The event horizon is inside of the region of a collapsed system, expecting a divergence due that $GM/R = 1/2$.

3. Discussion and conclusions

The evidence that we present in this paper foresees how is the blue-shift for an observer in an extremely no collapsed high curvature system, observing energy sources which come from

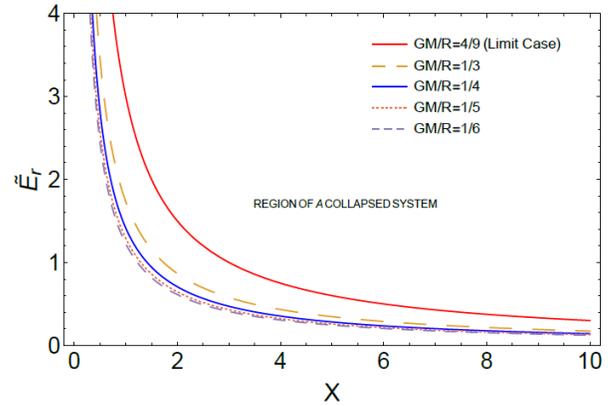


Fig. 2. The figure shows the increase of the received energy, varying the partner BH mass. Additionally, we have an energy increase while the ratio GM/R tends to the collapse value. Moreover, it is shown the limit region in where $GM/R = 4/9$ and the forbidden region called the region of collapse.

different phenomena such as CMB, T_{21} and Hawking radiation.

To describe the way that those photons, will behaves, we analyze the frame in their paths near a high curvature system and by determining their energy from the receiver point of view. Our analysis is under consideration that the observer (receiver) is in a non-collapsed system, being our analysis non valid for the particular region of the event horizon or within it.

We start with two well-known cosmological photons (CMB and T21 photons), describing the respectively equations to the energy in the receptor when the photons are subdued to system of high curvature. In this case, we consider that photons are decoupled from the Hubble flux, fixing the cosmological redshift to a particular era (z_f). Indeed, the energy increase for a non-collapsed system is limited as a maximum of three times of the energy of the source as it is shown in Fig. 1, however inside of a collapsed system, the energy bound does not exist, tending to infinity even on the Schwarzschild radius.

Another analysis is developed using the Hawking photons that comes from a partner BH. Our results confirm the general knowledge, that for a dwarf BH the Hawking radiation in the receptor tends to increase. But our results also show, how a system with a potential near to the expected for a collapse could increase substantially the Hawking radiation in the receptor. Moreover, we present the region of collapse in where the Schwarzschild radius is found.

As final comments, we emphasize that this study give us ideas of how photons behaves inside of a system with high curvature, like NS or other more exotic no collapsed objects. These results give us responses of how exterior radiation is increased near of an event horizon of a BH, having a bath of high energy photons from exterior sources. Our equations for CMB, T_{21} and Hawking radiation, can be easily extended to an arbitrarily distance to the event horizon but never in the frontier or within the BH; for this task should be necessary an appropriate change of coordinates like those proposed by Kruskal and Szekeres or for a time independent

coordinates like those proposed by Eddington and Finkelstein. Finally, a natural extension of this study can be developed through the addition of an energy-momentum tensor in the field equations, in order to have a crust of matter in the non-collapsed system and in particular, for a NS it is necessary a polytropic equation of state. However, this extension will be presented elsewhere.

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References

- [1] Palenzuela, C., L. Lehner, and Steven L. Liebling. "Orbital dynamics of binary boson star systems." *Physical Review D* 77.4, 2008, No. 4, Vol. 77.
- [2] Trümper, J. E., et al. "The puzzles of RX J1856. 5-3754: neutron star or quark star?" *Nuclear Physics B-Proceedings Supplements* 132, 2004, Vol. 132, p.560-565.
- [3] Abbott, Benjamin P., et al. "GW170817: observation of gravitational waves from a binary neutron star inspiral." *Physical Review Letters* 119.16, 2017, No.16, Vol. 119.
- [4] Event Horizon Telescope Collaboration. "First M87 event horizon telescope results. I. The shadow of the supermassive black hole." arXiv preprint arXiv:1906.11238, 2019.
- [5] Rivera-Paleo, F. J., and F. S. Guzmán. "CAFE-R: A Code That Solves the Special Relativistic Radiation Hydrodynamics Equations." *The Astrophysical Journal Supplement Series* 241.2, 2019, p. 28.
- [6] González-Avilés, J. J., et al. "I. Jet formation and evolution due to 3D magnetic reconnection." *The Astrophysical Journal* 856.2, 2018, No. 2, Vol. 856.
- [7] Penrose, Roger. "Gravitational collapse and space-time singularities." *Physical Review Letters* 14.3, 1965, No. 3, Vol. 14.
- [8] Aghanim, Nabila, et al. "Planck 2018 results-VI. Cosmological parameters." *Astronomy & Astrophysics* 641, 2020, Vol. 641.
- [9] Bowman, Judd D., et al. "An absorption profile centred at 78 megahertz in the sky-averaged spectrum." *Nature* 555.7694, 2018, No. 7694, Vol. 555.
- [10] Pound, Robert V., and Glen A. Rebka Jr. "Apparent weight of photons." *Physical Review Letters* 4.7, 1960, No. 7, Vol. 4.
- [11] Yang, Weiqiang, et al. "Dawn of the dark: unified dark sectors and the EDGES Cosmic Dawn 21-cm signal." *Journal of Cosmology and Astroparticle Physics* 2019.11, 2019, No. 11, Vol. 2019.
- [12] Hawking, Stephen W. "Black hole explosions?" *Nature* 248.5443, 1974, No. 5443, Vol. 248.
- [13] Weinberg, Steven. *Cosmology*. Oxford university press, 2008.